



Capacity estimation of lithium-ion battery with multi-task autoencoder and empirical mode decomposition

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ABSTRACT

Capacity estimation of lithium-ion batteries is a commonly used method in health diagnosis and management. Its mainstream method involves using data-driven time series forecasting models to learn the patterns of changes in capacity. However, capacity regeneration poses a challenge for training time series forecasting models. Therefore, we propose a hybrid method that applies empirical mode decomposition and a multi-task autoencoder. In detail, empirical mode decomposition is applied to decompose the time series of capacity into intrinsic mode functions and a residual. Then, a multi-task autoencoder based on diagonal state space models is applied to estimate intrinsic mode functions while support vector regression is utilized for the residual. Experimental results show that the method outperforms seven baselines on three datasets, with an average root mean square error of 0.0103, 0.0111, and 0.0004. Furthermore, it is capable of performing an inference on the CPU in 3.57 ms with 0.69 MB of memory usage.

1. Introduction

As the latest generation of high-performance batteries, lithium-ion batteries have outstanding advantages such as high voltage, high energy density, good cycling performance, low self-discharge, and no memory effect [1]. In recent years, lithium-ion batteries have undergone rapid development and have been widely used in industries such as electric vehicles. However, capacity degradation poses challenges to the application of lithium-ion batteries. The causes of this phenomenon include overcharging and self-discharge of batteries, dissolution of electrode materials, cathodic reduction and anodic oxidation of electrolyte components, as well as corrosion of current lead materials [2]. The industry of electric vehicles has high requirements for safety, and a decrease in battery capacity can increase the risk of malfunctions [3]. Therefore, accurately estimating battery capacity is crucial for health diagnosis and management in electric vehicles [4].

Current research on capacity estimation of lithium-ion batteries can be categorized into three types: model-based methods, data-driven methods, and hybrid methods [5]. The model-based method, which encompasses the physical-based model, equivalent circuit model, and filtering method, is employed to construct a physical model of the equipment's life cycle, taking into account the condition and failure mechanism [6,7]. This type of method starts with the degradation mechanism of lithium-ion batteries and therefore has high stability.

However, the model-based methods heavily rely on expert experiences and are only limited to specific conditions. In contrast, data-driven methods entail the extraction of sufficient feature information from raw data and the subsequent construction of a degradation model to estimate capacity [3,8]. This type of method can be realized by fast training with historical capacity data and transfer learning can be performed for different objects. However, data-driven methods are limited by the quality of training data and may suffer from high resource consumption. In contrast, the hybrid method combines two or more model-based or data-driven methods in order to enhance their respective strengths and overcome the limitations of models, thereby improving overall predictive performance [4,9,10]. Nevertheless, numerous methods forecast the original capacity sequence without accounting for uncertain changes in battery capacity resulting from regeneration.

In recent times, several studies have commenced the application of signal decomposition algorithms to the capacity estimation of lithium-ion batteries with the objective of achieving explicit modeling of the phenomenon of capacity regeneration [11–13]. In order to ensure the accuracy of capacity estimation, it is of the utmost importance to consider the uncertain changes caused by capacity regeneration. Signal decomposition algorithms are also employed in a variety of other time series forecasting tasks [14–16]. They permit the decomposition of uncertain changes resulting from a variety of causes, thereby enabling

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the explicit modeling of uncertain changes. Nevertheless, time series forecasting models may be less effective due to the quality of the data. Multi-task learning can facilitate the model's ability to learn a more generalized representation of the time series, thereby enhancing its time series forecasting capabilities [17].

Therefore, we present a hybrid method for capacity estimation of lithium-ion batteries that integrates empirical mode decomposition (EMD), a multi-task autoencoder (MTAE), and support vector regression (SVR). Notably, the empirical mode decomposition is a method that can decompose a time series into multiple intrinsic mode functions (IMFs) and a residual. In detail, the IMFs represent the uncertain changes caused by capacity regeneration, while the residual represents the overall trend. Subsequently, a novel multi-task autoencoder based on diagonal state space models is employed to fit the intrinsic mode functions, in conjunction with support vector regression for the residual. Finally, the predictions from both components are combined to form a capacity estimation of lithium-ion batteries.

Experiments were conducted on three publicly available battery datasets: NASA, CALCE, and Toyota. The experimental results demonstrate that the proposed method outperforms other baselines on all three datasets. Compared to TiDE [18], the proposed method achieves an average reduction of 0.0056, 0.0229, and 0.006 in the root mean square error, with a remarkable reduction of 0.5065 in the mean absolute percentage error on the Toyota battery dataset. Moreover, the proposed hybrid method achieves an average root mean square error of 0.0103, 0.0111, and 0.0004 on three datasets. Notably, it can perform an inference on the CPU in 3.57 ms with 0.69 MB of memory usage. These results demonstrate that the proposed method accurately estimates the capacity of lithium-ion batteries, which can assist in ensuring their safety.

The main contributions of this paper are as follows:

- (1) In order to eliminate any uncertain changes associated with capacity regeneration, empirical mode decomposition is employed to explicitly decompose the time series of battery capacity.
- (2) A novel multi-task autoencoder based on diagonal state space models is proposed, which enhances the capability of time series forecasting by incorporating an additional time series reconstruction task.
- (3) In order to accurately estimate the battery capacity in the presence of uncertain changes caused by capacity regeneration, multi-task autoencoders and support vector regression are applied to predict the intrinsic mode functions and residual, respectively.
- (4) Experimental results demonstrate that the proposed hybrid method exhibits the best performance on three datasets while taking up fewer resources and having high real-time efficiency.

The rest of the paper is organized as follows. Section 2 reviews some related literature and work. Section 3 introduces the hybrid method. Section 4 presents the experimental results along with illustrative descriptions. Finally, the conclusion is expounded in Section 5.

2. Related work

2.1. Model-based methods

Model-based capacity estimation of batteries refers to the construction of a degradation model for battery capacity based on electrochemical principles, enabling capacity estimation from a chemical mechanism perspective. Bartlett et al. proposed a reduced-order electrochemical model for a composite electrode battery that predicts the surface and bulk lithium concentrations of each material, which is then used in a dual-nonlinear observer to estimate the battery capacity [6]. Similarly, Allam et al. proposed an enhanced single particle model that utilizes the relationship between solid electrolyte and power attenuation to

achieve a combined estimation of lithium concentration, battery capacity, and aging-sensitive transport parameters in the electrode [7]. However, these methods rely on a large number of experiments and expert experiences to determine the model's parameters and require the establishment of a new model for new materials. Moreover, capacity estimation based on electrochemical models requires complex calculations and numerical simulations, demanding high computational costs.

2.2. Data-driven methods

Data-driven capacity estimation of batteries refers to the method of training a data-driven model using a time series of historical capacity data to predict future capacity. Many classic machine learning models have been applied to battery capacity estimation, including support vector regression (SVR) [19–21], gaussian process regression (GPR) [22–24], and random forest (RF) [25], among others. These traditional machine learning methods have low computational costs so that they can perform real-time capacity estimation. However, they are weak for mining potential features and therefore cannot accurately predict battery capacity in the case of capacity regeneration. With the rise of deep learning, an increasing number of scholars are applying deep learning models to the capacity estimation of batteries. Zhang et al. proposed a deep learning method that uses long short-term memory (LSTM) recurrent neural network (RNN) to learn long-term dependencies [3]. Similarly, Mamo et al. added the tension mechanism to the long short-term memory recurrent neural network and used the attention mechanism to solve the problem of long-term memory forgetting [26]. Wang et al. proposed a bidirectional long short-term memory with an attention mechanism to predict online capacity by continuously updating the model's parameters [27]. However, these methods directly model and predict the capacity sequence without considering the uncertain changes caused by capacity regeneration. Capacity regeneration affects the quality of training data, thus making it difficult to train high-quality time series forecasting models. Therefore, not explicitly modeling capacity regeneration would affect the final prediction accuracy and result in inaccurate capacity estimation.

2.3. Hybrid methods

In order to capitalize on the strengths of multiple models while mitigating their weaknesses, numerous scholars have initiated research into hybrid methods [28,29]. Hybrid methods refer to the combination of two or more model-based or data-driven methods to enhance overall predictive performance. The current mainstream trend is to estimate battery capacity using hybrid methods, primarily by combining signal decomposition algorithms and time series forecasting models. Qu et al. presented a neural network-based method that combines a long short-term memory network with the attention mechanism and Complete Ensemble Empirical Mode Decomposition with Adaptive Noise [30]. Similarly, Li et al. proposed a hybrid Elman-LSTM method for estimating battery capacity by combining the empirical mode decomposition algorithm and long short-term memory and Elman neural networks [4]. Pang et al. proposed a novel method fusing the wavelet decomposition technology (WDT) and the Nonlinear Auto Regressive Neural Network (NARNN) model for estimating the capacity of lithium-ion batteries [31]. These methods integrate classical time series forecasting models with signal decomposition algorithms to explicitly model uncertain changes due to capacity regeneration. Similar hybrid methods have been employed in other time series forecasting tasks. Lotfipour et al. proposed a hybrid method that combined empirical mode decomposition with a convolutional neural network and long short-term memory to forecast short-term residential loads [14]. Similarly, Moreno et al. combined variational mode decomposition with autoregressive recurrent neural networks to forecast wind speed [15]. Although these methods combine signal decomposition algorithms with time series

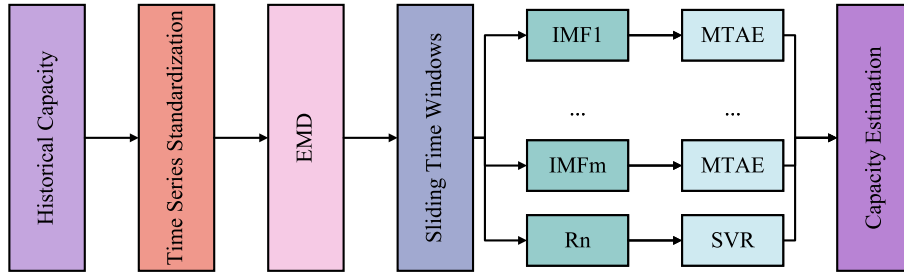


Fig. 1. The proposed framework for capacity estimation of lithium-ion batteries.

forecasting models, the employed time series models still have relatively weak capabilities. The time series forecasting models employed in these methods are focused on the forecasting task and still have high requirements for data quality. In contrast, multi-task learning allows the model to learn a more generalized representation of the time series to enhance its performance on time series forecasting tasks. Recently, state space models have shown to be extremely competitive in several sequence modeling tasks [32–34]. Therefore, we creatively combined them for the capacity estimation of lithium-ion batteries.

3. Methodology

To accurately estimate battery capacity under uncertainty caused by capacity regeneration, a hybrid method combining empirical mode decomposition, a multi-task autoencoder, and support vector regression is proposed in this study, as shown in Fig. 1. First, we preprocess the capacity time series, including time series standardization, empirical mode decomposition, and sliding time windows. In detail, we decompose the capacity time series using empirical mode decomposition to obtain intrinsic mode functions and a residual. Subsequently, a novel multi-task autoencoder based on diagonal state space models is employed to fit intrinsic mode functions, while the support vector regression is utilized for a residual. The final step in the process is to combine the two parts' predictions to arrive at a capacity estimation. Next, we will provide a detailed description of the techniques used and eventually detail how they can be combined for capacity estimation of lithium-ion batteries.

3.1. Empirical mode decomposition

To eliminate uncertainty caused by capacity regeneration, empirical mode decomposition [35] is applied to explicitly decompose the capacity time series into multiple intrinsic mode functions (IMFs) and a residual. The intrinsic mode functions represent the uncertain changes brought by capacity regeneration, while the residual represents the overall trend. By summing these components, the capacity time series can be reconstructed. The implementation steps of empirical mode decomposition are as follows.

The first step is to identify all local maxima and minima of the original signal $w(t)$, and then employ the cubic spline interpolation method to generate the upper and lower envelopes $l_1(t)$ and $l_2(t)$. The mean envelope $m_1(t)$ is then calculated as follows:

$$m_1(t) = \frac{l_1(t) + l_2(t)}{2} \quad (1)$$

The first IMF component $h_1(t)$ can be obtained by removing $m_1(t)$ from the original signal $w(t)$.

$$h_1(t) = w(t) - m_1(t) \quad (2)$$

If $h_1(t)$ does not satisfy the conditions of the IMF component, then $h_1(t)$ is regarded as a new $w(t)$, and the previous two steps are repeated. Assuming that the conditions are satisfied after k iterations of the calculation process, the first IMF component is calculated as follows:

$$h_{1k}(t) = h_{1_{k-1}}(t) - m_{1k}(t) \quad (3)$$

The first IMF component $h_1(t)$ is extracted from the original signal $w(t)$, and then the aforementioned procedure is repeated until the final remaining portion exhibits a monotonic or constant value sequence. The residual is then defined as the remaining portion. After decomposition, the original signal $w(t)$ can be reconstructed by summing all IMF components $h_i(t)$ and the residual $r_n(t)$ as follows:

$$w(t) = \sum_{i=1}^N h_i(t) + r_n(t) \quad (4)$$

where N denotes the number of IMFs. Therefore, IMF components and the residual can be separately predicted, and then all the predictions can be combined to reconstruct the prediction of the original signal.

Fig. 2 displays the results of the empirical mode decomposition applied to the time series of three batteries. The IMFs are regular oscillation curves, exhibiting a lower degree of oscillation than the original time series. This makes them more amenable to modeling and prediction. However, the temporal dependency of their changes is still very complex, so we apply the multi-task autoencoder based on diagonal state space models with strong extraction capability of temporal dependency to predict IMFs. In addition, it can be found that the residuals capture the general trend of the capacity time series well. Compared to the raw data, the residuals are smoother and show a monotonic trend. Therefore, we simply apply support vector regression to predict the residual.

3.2. Multi-task autoencoder

Fig. 3 depicts the model architecture of the proposed multi-task autoencoder, which comprises the diagonal state space model S4D and multi-layer perceptrons (MLPs) with residual connections. Multi-layer perceptrons are applied to aggregate spatial dependencies between different features, and the diagonal state space model for temporal dependencies while keeping the input dimension unchanged. The model is capable of rapid convergence due to the transfer of features across layers facilitated by residual connections, which enhances prediction and reconstruction capabilities. Furthermore, through multi-task learning, the model can learn a more generalized representation of the time series, thereby improving the accuracy of the predictions. The subsequent section will provide a detailed account of two of the most significant modules.

Diagonal state space model. The state space model (SSM) has recently demonstrated superior performance in numerous sequence modeling tasks. Consequently, we have devised a novel method to apply it to the construction of a multi-task autoencoder. Following the guidance of [32], S4D has been chosen as the foundation for the state space model layer, which possesses strong mining abilities of temporal dependencies. The essence of S4D is a one-dimensional convolutional neural network, which is empowered by complex calculations using a diagonal state space, allowing it to extract long-range temporal dependencies. The calculation of a state space model can be represented as follows:

$$u'(t) = Au(t) + Bx(t) \quad (5)$$

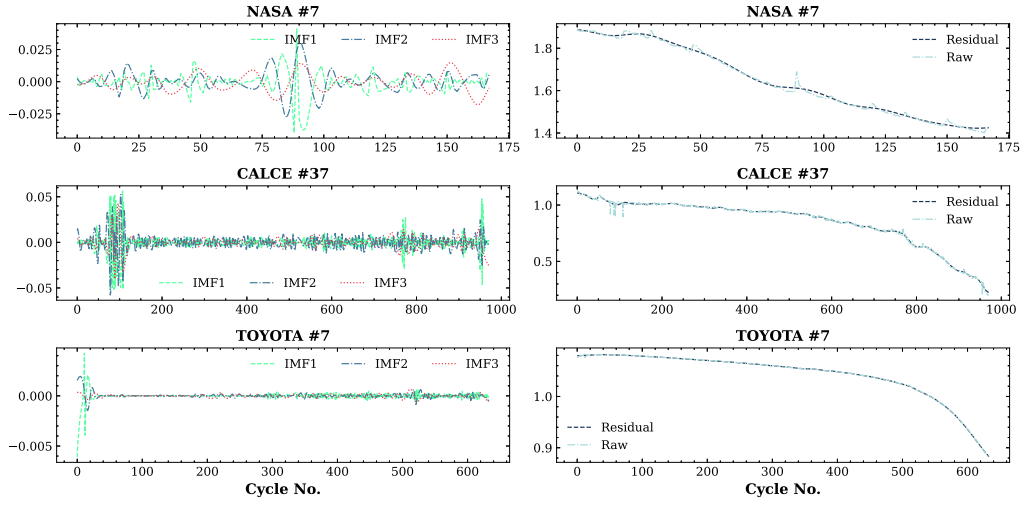


Fig. 2. The results of three capacity time series after empirical mode decomposition.

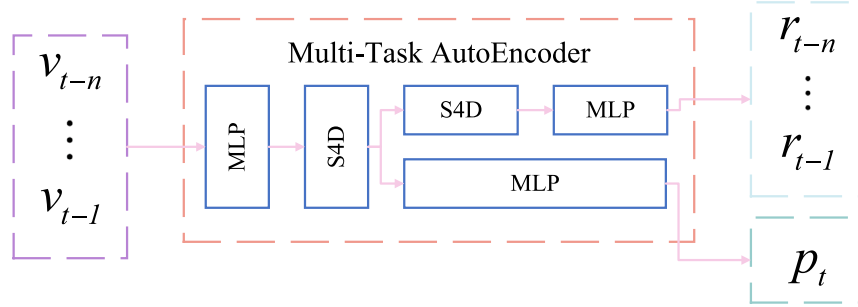


Fig. 3. The architecture of the multi-task autoencoder.

$$y(t) = Cu(t) \quad (6)$$

where $A \in \mathbb{R}^{n \times n}$ and $B, C \in \mathbb{R}^n$ are state matrices, and $u(t) \in \mathbb{R}^n$ is the latent state, and $x(t)$ denotes the input and $y(t)$ denotes the output. In S4D, A is a diagonal matrix associated with the diagonal state space. S4D benefits from a convolutional kernel generated from the parameters of a diagonal state space model, which is more robust compared to a convolutional neural network with random initialization of the parameters. Let u denote the input of S4D, k denote the kernel generated from the parameters of a diagonal state space model, and the convolution operation is performed first.

$$c = Conv(u, k) \quad (7)$$

where $Conv$ denotes the convolution operation and c is the output. Then, a vector D is used to transform u and added to c to implement the residual connection. In addition, the nonlinear activation function $GELU$ is used. To prevent overfitting, dropout is added.

$$d = Dropout(GELU(c + u \cdot D)) \quad (8)$$

Finally, one-dimensional convolution is applied to keep the output in the same dimension as the input of the S4D. Moreover, the activation function GLU is used.

$$e = GLU(Conv1d(d)) \quad (9)$$

It is worth noting that the input u and the output e of S4D have the same dimensions. Due to capacity regeneration, accurate prediction of uncertain changes in battery capacity is the key problem to capacity estimation. With S4D, it is possible to capture the temporal dependency of the capacity time series and thus accurately predict IMFs. Therefore, we propose a novel multi-task autoencoder based on S4D for estimating battery capacity.

Multi-layer perceptron. A multi-layer perceptron is a neural network comprising multiple fully connected layers connected in series. Non-linear transformations are achieved between every two fully connected layers through the activation function. It can uncover spatial dependencies between different features and theoretically can approximate any function. However, the fully connected layer in a multi-layer perceptron is prone to overfitting on the training dataset, and the model training is relatively slow due to the large number of parameters. Therefore, residual connections are introduced into the fully connected layer, similar to [36], and dropout is added to achieve fast training of the model and mitigate overfitting. Let x denote the input of multi-layer perceptron, the calculation process is as follows:

$$h_1 = GELU(xW_1^T + b_1) \quad (10)$$

$$h_2 = h_1W_2^T + b_2 \quad (11)$$

$$h_3 = Dropout(h_2) + xW_3^T + b_3 \quad (12)$$

$$o = LayerNorm(h_3) \quad (13)$$

where W_1, W_2, W_3 are weights and b_1, b_2, b_3 are biases. h_1, h_2 , and h_3 are outputs of three fully connected layers and o is the output of the multi-layer perceptron. $LayerNorm$ is applied to limit the output to a specified range to improve the training speed and model's performance. The multi-layer perceptron is capable of efficiently performing non-linear transformations that can allow the model to integrate different features, resulting in more complex features to enhance the model's representational capabilities. It is worth noting that the multi-layer perceptron used for dimension conversion of outputs does not use layer normalization.

Encoder-shared training. The multi-task autoencoder employs a single encoder to perform both time series forecasting and reconstruction. During the training phase, the multi-task autoencoder simultaneously reconstructs and predicts the input time series. Let \mathcal{L}_{pre} denote the prediction loss, \mathcal{L}_{rec} denote the reconstruction loss, and the total loss \mathcal{L}_{total} is obtained by their linearly weighted summation.

$$\mathcal{L}_{total} = \alpha \cdot \mathcal{L}_{pre} + \beta \cdot \mathcal{L}_{rec} \quad (14)$$

where α and β are the weights of the prediction loss and the reconstruction loss, to control the attention of the multi-task autoencoder to the specific task. Let $\{v_{t-n}, \dots, v_{t-1}, v_t\}$ denotes the input time series, $\{r_{t-n}, r_{t-n+1}, \dots, r_{t-1}\}$ denotes the reconstruction, and $\{p_t\}$ denotes the prediction, \mathcal{L}_{pre} and \mathcal{L}_{rec} is calculated as follows:

$$\mathcal{L}_{pre} = |v_t - p_t| \quad (15)$$

$$\mathcal{L}_{rec} = \frac{1}{n} \sum_{i=1}^n |v_{t-i} - r_{t-i}| \quad (16)$$

The process of multi-task learning necessitates the simultaneous optimization of time series forecasting and the reconstruction of the objective. Consequently, a more generalized representation of the time series is learned in comparison to a single task [37]. As in [17], the researchers applied multi-task learning to time series forecasting and reconstruction, and ultimately to anomaly detection in multivariate time series. A loss function analogous to that employed by the aforementioned researchers is employed in order to simultaneously optimize both the time series forecasting and reconstruction tasks.

3.3. Support vector regression

Support vector regression (SVR) is a frequently utilized regression model that is capable of fitting various nonlinear functions with the assistance of multiple kernel functions. In this study, the residual obtained after empirical mode decomposition is a relatively smooth nonlinear curve. Therefore, we apply support vector regression to predict it. The use of support vectors allows support vector regression to demonstrate a high degree of tolerance for outliers and to capture overall trends with great efficacy. Consequently, support vector regression is an effective method for fitting the residual, thereby enabling the accurate prediction of the overall trend. Support vector regression is a method that combines the concept of support vector machines to solve linear regression problems, which is achieved by utilizing the principle of support vectors. The objective of support vector regression is to optimize the following:

$$\min_{w,b} \frac{1}{2} \|w\|_2^2 \quad (17)$$

where w is the weight vector of the support vector regression line and b is its corresponding bias term. The points situated within the margin satisfy the following conditions:

$$|y_i - (w^T x_i + b)| \leq \epsilon \quad (18)$$

where (x_i, y_i) denotes the i th point and ϵ is a threshold. The cost function for support vector regression is as follows:

$$\frac{1}{n} \sum_{i=1}^n l_\epsilon(x_i, y_i) \quad (19)$$

$$l_\epsilon(x_i, y_i) = \max(0, 1 - y_i(w^T x_i - b)) \quad (20)$$

where n is the number of data points. The final support vector regression optimization problem can be written as:

$$\min_{w,b} \frac{1}{2} \|w\|_2^2 + \frac{C}{n} \sum_{i=1}^n l_\epsilon(x_i, y_i) \quad (21)$$

where C is a hyper-parameter that controls the importance of the second item. In this task, the linear kernel is selected as the kernel function for support vector regression due to the relatively smooth residual and the presence of a linear trend.

3.4. Hybrid method

The proposed hybrid method can be primarily divided into three steps, as illustrated in Fig. 1. Prior to any further analysis, it is essential to preprocess the original capacity time series of batteries. This involves standardization, empirical mode decomposition, and sliding time window processing. Data preprocessing is crucial for the vast majority of machine learning tasks and its importance cannot be overstated. In order for the model to be trained in a timely and accurate manner, it is necessary to standardize the capacity time series. This process allows the model to learn the changing patterns of the capacity time series more effectively. Standardization enables all values to be scaled to a narrow interval, thus eliminating the effect of ranges of variable value on data processing. Let x denote the capacity time series, μ denote the average value of the capacity time series, and σ denote the standard deviation, then standardized time series \hat{x} is calculated as follows:

$$\hat{x} = \left\{ \frac{x_i - \mu}{\sigma}, i \in [1, n] \right\} \quad (22)$$

where n is the number of data points. Subsequently, the original capacity time series is decomposed into multiple high-frequency intrinsic mode functions and a low-frequency residual. In this study, we restrict the number of IMFs obtained from the empirical mode decomposition to a maximum of three. The time series also needs to be processed into input and output pairs before modeling for forecasting as Fig. 3. A sliding time window slices the time series, thus allowing the model to learn the temporal dependencies in a small segment of the time series. Then, multi-task autoencoders and support vector regression are applied separately to model the IMFs and residual. Finally, the predictions from both parts are summed to obtain the estimation of battery capacity.

4. Experiments

4.1. Experimental setup

Datasets. Three commonly used datasets NASA [38], CALCE, and Toyota [39] were selected as the datasets for this study. These datasets collectively include the historical capacity time series of 12 batteries, as shown in Fig. 4. First, we converted them to the form of capacity, which is measured in Ah. In this study, we use the first 70% of points in the NASA dataset and the first 85% of points in another two datasets to train the model and the rest to test the model's performance.

Baselines. Three classical nonlinear models were chosen as our baseline for the study: support vector regression (SVR), gaussian process regression (GPR), and multi-layer perceptron (MLP). In addition, TiDE [18] and DLinear [40] were included in our selection for comparison, as they have recently demonstrated strong capabilities in time series forecasting tasks. Moreover, To demonstrate the excellent performance of the multi-task autoencoder, we also compare it with two hybrid methods [9,28] based on signal decomposition algorithms. For these seven baselines, evaluations were conducted on three datasets as well.

Settings and implementation. The experiments were conducted on a workstation equipped with a single RTX 4090 GPU, an Intel Xeon Platinum 8336C processor running at 2.3 GHz, and 128 GB of memory. The experiments were implemented in Python 3.9 and the deep learning models were built using Pytorch 2.1. The training phase employed a learning rate decay strategy with $lr_decay = 0.99$. AdamW was used as the optimizer with $betas = (0.9, 0.98)$.

Evaluation metrics. In accordance with previous studies [13,27], the root mean square error (RMSE) and mean absolute percentage error (MAPE) were utilized to evaluate the model's performance. They are defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (x_i - \hat{x}_i)^2}{n}} \quad (23)$$

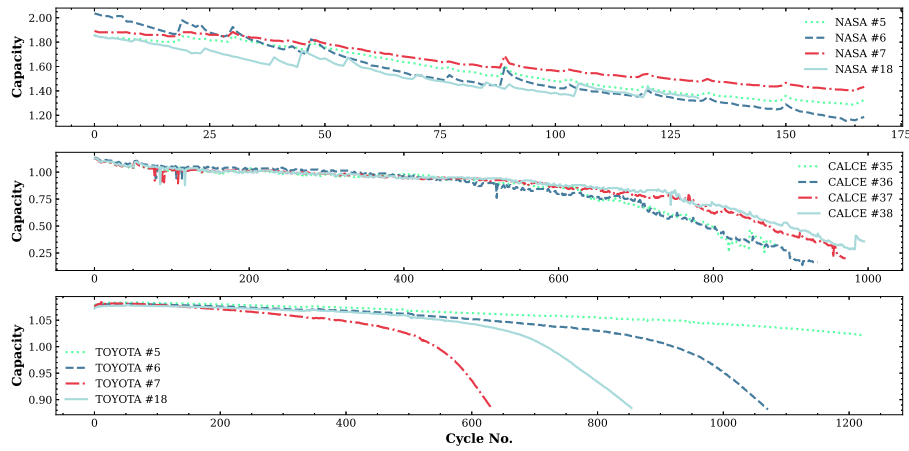


Fig. 4. The capacity time series of 12 lithium-ion batteries.

Table 1
Comparison of performance with different methods on three datasets.

NASA battery dataset								
Method	#5		#6		#7		#18	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
SVR	0.0106	0.6946	0.0129	0.6465	0.0087	0.4209	0.0246	0.8394
GPR	0.0140	0.6646	0.0134	0.8076	0.0086	0.3372	0.0232	0.9027
MLP	0.0117	0.7267	0.0140	0.8694	0.0125	0.5598	0.0289	1.7648
DLinear	0.0112	0.6550	0.0322	2.2928	0.0106	0.6431	0.0205	1.2019
TiDE	0.0119	0.6970	0.0160	0.9831	0.0095	0.4747	0.0263	1.3199
GPR-LSTM	0.0104	0.5944	0.0105	0.6391	0.0072	0.3685	0.0154	0.8958
CEEMDAN-GARG	0.0092	0.5956	0.0117	0.6617	0.0360	2.2515	0.0237	1.3203
MTAE-SVR	0.0089	0.5817	0.0098	0.6066	0.0059	0.2665	0.0166	0.7639
CALCE battery dataset								
Method	#35		#36		#37		#38	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
SVR	0.0217	3.2364	0.0134	2.6691	0.0138	2.0660	0.0167	1.8349
GPR	0.0207	2.7377	0.0150	2.9458	0.0158	2.6063	0.0173	2.3563
MLP	0.0756	11.2977	0.0279	6.8143	0.0220	3.5924	0.0221	2.7098
DLinear	0.0389	5.9191	0.0332	6.7790	0.0281	4.6164	0.0355	5.5075
TiDE	0.0371	5.3924	0.0329	7.1217	0.0271	4.3988	0.0388	5.0802
GPR-LSTM	0.0299	5.9228	0.0131	4.1931	0.0123	2.4633	0.0133	2.3280
CEEMDAN-GARG	0.1138	25.7013	0.0559	20.4090	0.0498	12.5890	0.0262	4.2552
MTAE-SVR	0.0140	1.7999	0.0102	2.2121	0.0090	1.2034	0.0110	1.0677
TOYOTA battery dataset								
Method	#5		#6		#7		#18	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
SVR	0.0005	0.0361	0.0007	0.0611	0.0008	0.0691	0.0019	0.1791
GPR	0.0005	0.0338	0.0019	0.1473	0.0044	0.3637	0.0019	0.1661
MLP	0.0009	0.0735	0.0028	0.2188	0.0124	0.9493	0.0062	0.5733
DLinear	0.0008	0.0581	0.0060	0.4700	0.0100	0.8395	0.0032	0.2573
TiDE	0.0008	0.0601	0.0011	0.0929	0.0023	0.1610	0.0213	1.8395
GPR-LSTM	0.0006	0.0480	0.0078	0.5901	0.0097	0.6801	0.0088	0.6124
CEEMDAN-GARG	0.0008	0.0531	0.0248	1.9115	0.0198	1.5002	0.0073	0.6303
MTAE-SVR	0.0004	0.0321	0.0003	0.0259	0.0004	0.0325	0.0004	0.0372

$$MAPE = \frac{100}{n} \cdot \sum_{i=1}^n \left| \frac{x_i - \hat{x}_i}{x_i} \right| \quad (24)$$

4.2. Performance comparison

The performance of the proposed hybrid method for estimating the capacity of lithium-ion batteries on three datasets is shown in Table 1. It can be seen that the proposed method achieves the best performance on all 12 battery capacity time series. The proposed method achieves an average root mean square error of 0.0103, 0.0111, and 0.0004 on three datasets when explicitly modeling capacity regeneration. It indicates

that the proposed method is more suitable for the capacity estimation of batteries with a more powerful time series forecasting capability.

Disappointingly, both TiDE and DLinear demonstrate poor performance on three datasets. Under the influence of capacity regeneration, the capacity time series exhibits complex variations, making it difficult for time series forecasting models to effectively learn its patterns. The proposed hybrid method can explicitly model capacity regeneration and thus achieves excellent performance. Although the first three models are simple, they demonstrate strong competitiveness, especially support vector regression. However, compared to the proposed method, its performance is still poor, which also benefited from the explicit modeling of capacity regeneration brought by empirical mode decomposition.

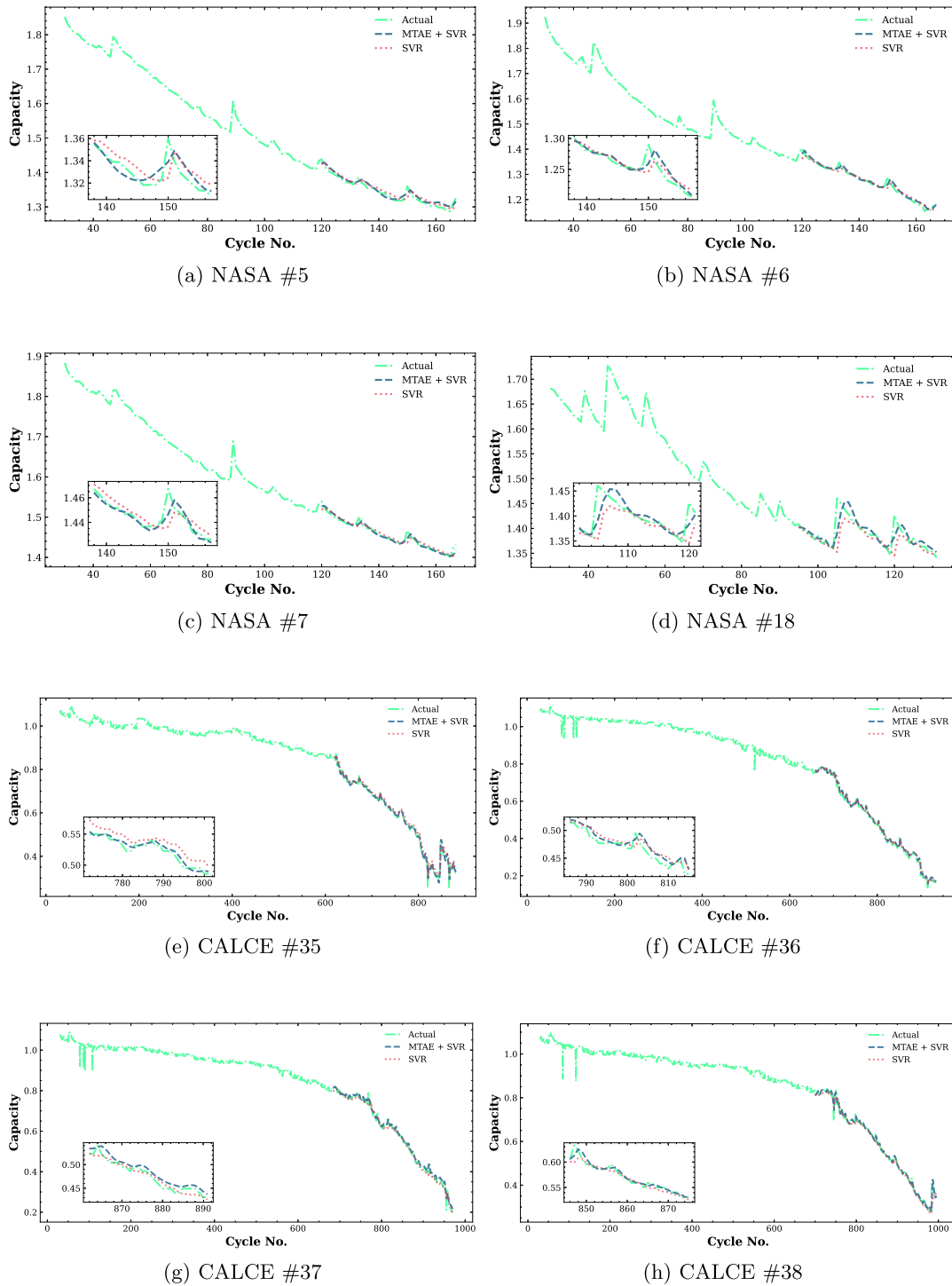


Fig. 5. The predictions of different methods on two datasets.

Compared with two hybrid methods based on signal decomposition algorithms, we find the excellence of the proposed method. It can be found that the two methods do not perform well on some batteries. We found that CEEMDAN-GARG performed well only on some batteries. It uses ARIMA to fit the residual, which is sensitive to the choice of hyper-parameters. Moreover, ARIMA does not fit the residuals of some of these capacity time series well and requires extensive attempts at hyper-parameters. CEEMDAN-GARG achieves poorer performance because the residuals represent the general trend of capacity. However, the GRU model in the method fits the IMFs well, which illustrates

the effectiveness of recurrent neural networks for modeling IMFs. In contrast, the hybrid GPR-LSTM method also did not show strong performance on all batteries due to the poor estimation of IMFs by GPR. In addition, we observed that LSTM does not show a good performance for the estimation of residuals. Moreover, the models integrated in these hybrid methods are sensitive to the selection of hyper-parameters.

Fig. 5 shows the performance of the proposed method, as well as the performance of SVR and GPR on two datasets. It can be found that the proposed method has a stronger ability to capture uncertain changes caused by capacity regeneration compared to the other two

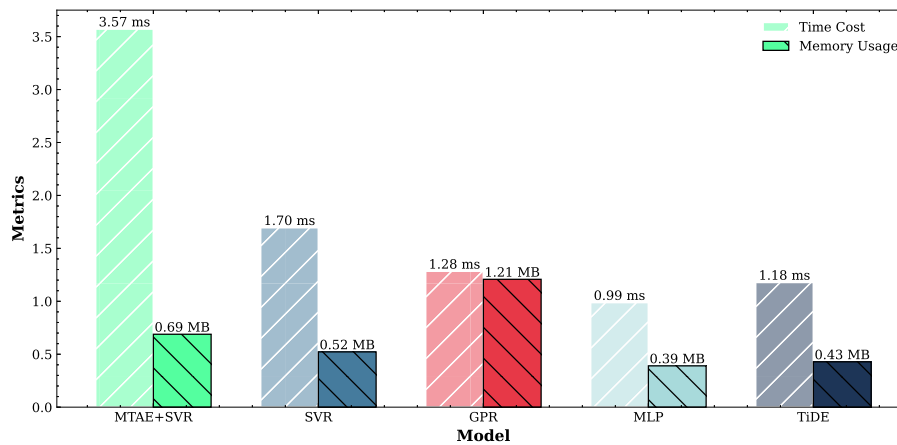


Fig. 6. The time and memory required for inference of different methods.

Table 2

Performance of different model combinations on CALCE #37 battery.

Method	MTAE		LSTM		GPR		SVR	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
GPR	0.0116	1.9192	0.0122	2.0291	0.0115	1.7757	0.0104	1.5785
SVR	0.0090	1.2034	0.0096	1.4099	0.0094	1.2626	0.0115	1.7748

methods. Therefore, the proposed method is more suitable for estimating the capacity of lithium-ion batteries and can provide more accurate predictions.

Table 2 shows the performance of different model combinations when applied to the CALCE dataset #37 battery. Each of the four models in the horizontal direction was used to predict the IMFs, and the two models in the vertical direction were used to predict the residual. It can be found that the combination of MTAE and SVR achieved the best performance, which illustrates the excellence of the proposed hybrid method. Notably, using SVR for residual prediction gives significantly better results than GPR. However, SVR is less effective in predicting IMFs, so the GPR-SVR combination in the last column exceeds the results of using SVR alone.

In addition, we tested the time cost and memory usage of some of the methods. As shown in Fig. 6, all methods are able to achieve ms-level operation speeds, and the proposed hybrid method operates at an average of 3.57 ms on the CPU. Although significantly more time-consuming than other models, it is still minimal enough to do real-time estimation of lithium-ion battery capacity. It is worth noting that this is the computational cost of single-thread inference. If it is a four-thread inference, the required inference time will be shortened. It can be found that the proposed method requires only 0.17 MB more memory and 1.87 ms more time consumption compared to the solo SVR model, which indicates that the proposed method requires a small computational cost while ensuring the accuracy of capacity estimation of lithium-ion batteries.

4.3. Ablation study

To test the effectiveness of some modules in the proposed method, ablation experiments were conducted. In this section, the effects of multi-task learning and empirical mode decomposition will be explored.

Effect of multi-task learning. The proposed multi-task autoencoder simultaneously performs two tasks: time series forecasting and reconstruction. Fig. 7 shows the results of ablation experiments on the reconstruction task of the multi-task autoencoder on the CALCE dataset. It can be seen that benefiting from multi-task learning, the proposed

method performs better with the addition of a reconstruction task than without. The two tasks share the encoder, so the multi-task autoencoder can learn a better time series representation.

Effect of EMD. Empirical mode decomposition is important as a key technique for explicitly modeling battery capacity regeneration. So the results of whether or not to use empirical mode decomposition were compared, as shown in Fig. 8. The performance of using only the support vector regression and the proposed hybrid method was compared using the NASA dataset as an example. It can be found that with the addition of empirical mode decomposition, the model is more inclined to naturally predict the irregular changes brought about by capacity regeneration. Therefore, the proposed method is more suitable for predicting the battery capacity in the case of battery capacity regeneration.

4.4. Parameter sensitivity

The choice of hyper-parameters is crucial for deep learning models. In this section, the sensitivity of important hyper-parameters will be explored. The sliding time window size and loss weights significantly affect the multi-task autoencoder and were selected for sensitivity analysis.

Window size. The choice of sliding window size is crucial for time series forecasting. Therefore, ten different values were experimented with for battery #37 of the CALCE dataset, as shown in Fig. 9. It can be seen that the optimal window size is ten, and either too large or too small will result in poor model performance. However, the overall performance of the model is still better, which indicates the robustness of the proposed method.

Loss weights. The assignment of loss weights in the multi-task autoencoder affects the model's focus on each task. To explore the specific effect of loss weights on model performance, 25 sets of loss weights were selected for the study. As shown in Fig. 10, it can be seen that when α is greater than β , the multi-task autoencoder tends to focus more on the prediction task and thus achieves a lower root mean square error of prediction. The multi-task autoencoder is robust to the selection of loss weights as long as α is larger than β .

5. Conclusion

To accurately estimate the capacity of lithium-ion batteries under capacity regeneration, we propose a hybrid method that utilizes a multi-task autoencoder and empirical mode decomposition. With the support of the empirical mode decomposition, excellent capacity estimation performance has been achieved by explicit modeling of capacity

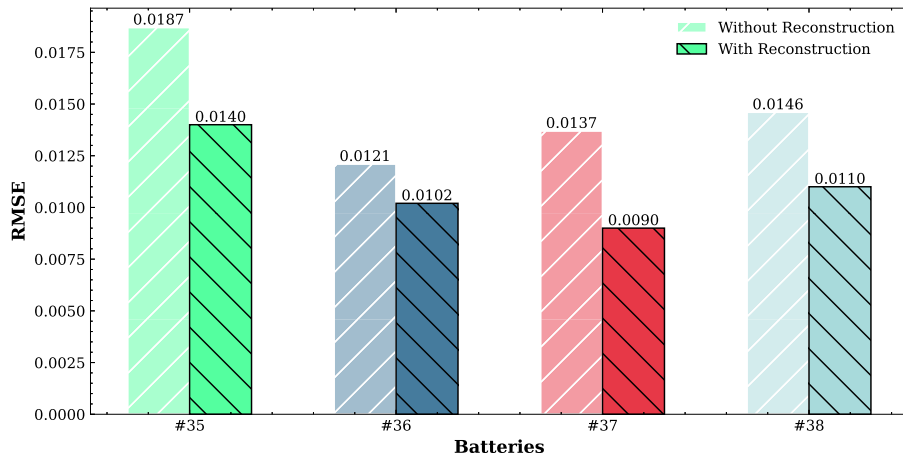


Fig. 7. The results of ablation experiments on the reconstruction task of the multi-task autoencoder on the CALCE dataset.

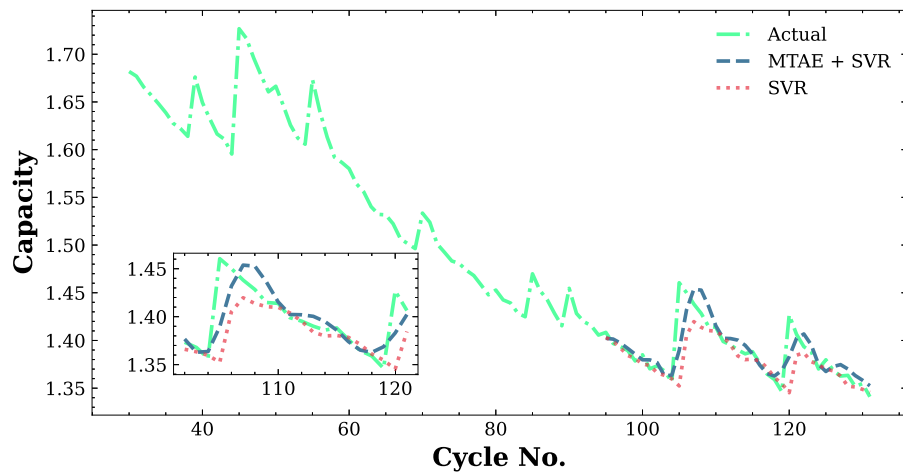


Fig. 8. The results of ablation experiments of whether using EMD on the NASA dataset battery #18.

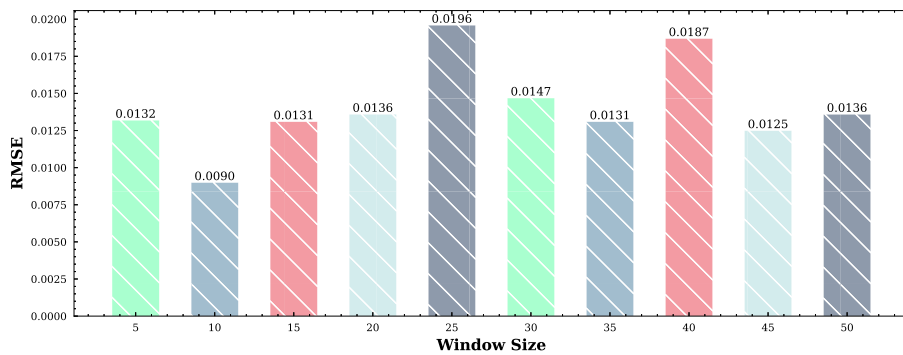


Fig. 9. The performance of the proposed method under different sliding window sizes.

regeneration. In the wave of multi-task learning, a novel diagonal state space model-based multi-task autoencoder is proposed to more accurately predict the changes caused by capacity regeneration. The experiments were conducted on three public datasets, and the proposed method outperformed the five baselines. Experimental results show that the proposed method can eliminate the uncertainty caused by capacity regeneration, accurately estimating the capacity of lithium-ion batteries. Therefore, the proposed method can provide a more accurate estimation of lithium-ion battery capacity, ensuring its safety. However,

whether the proposed method can accurately estimate battery capacity in actual scenarios has not yet been tested. There are limitations to conducting experiments on only three public datasets. In addition, the proposed method is based on univariate time series forecasting, whereas complex spatial dependencies between multivariate variables can provide more information for the capacity estimation of lithium-ion batteries. In the future, we will try to apply the proposed method to real scenarios for testing. In addition, the impact of different signal decomposition algorithms and the potential of state space models will

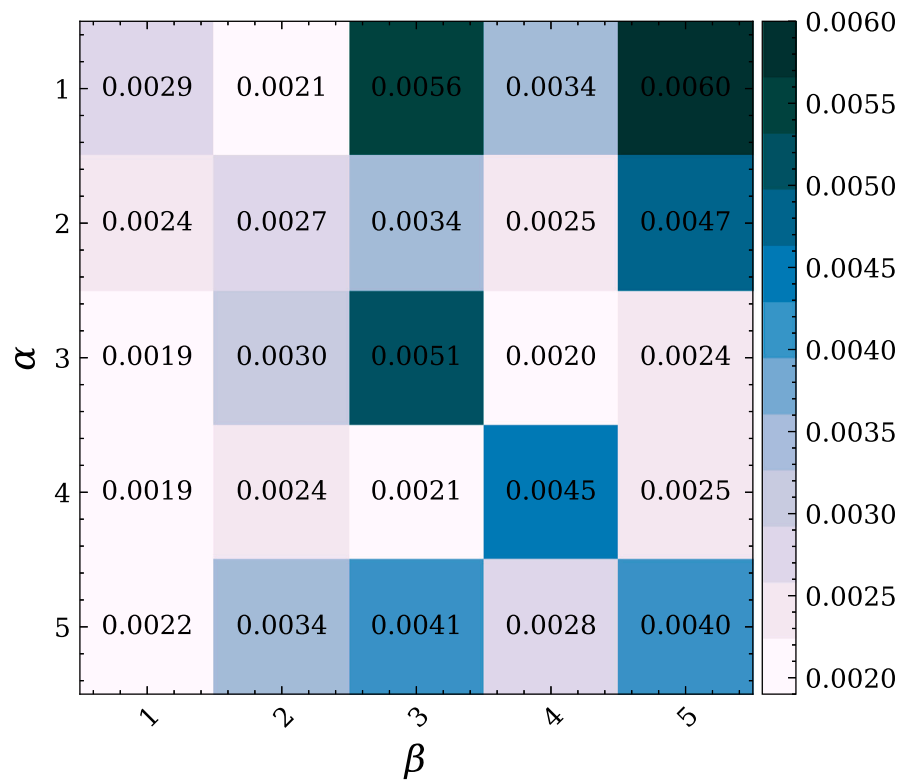


Fig. 10. The performance of a multi-task autoencoder for IMF_3 under different loss weights.

be further explored. Most importantly, we will attempt to predict the capacity of lithium-ion batteries based on a multivariate time series forecasting method.

CRedit authorship contribution statement

Qi Sun: Writing – original draft, Visualization, Validation, Methodology, Investigation. **Fangshu Cui:** Writing – review & editing, Supervision, Investigation. **Mingrui Shi:** Writing – review & editing, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors are unable or have chosen not to specify which data has been used.

References

- [1] Xinghui Zhang, Zhao Li, Lingai Luo, Yilin Fan, Zhengyu Du, A review on thermal management of lithium-ion batteries for electric vehicles, *Energy* 238 (2022) 121652.
- [2] L.S. Kanevskii, V.S. Dubasova, Degradation of lithium-ion batteries and how to fight it: A review, *Russ. J. Electrochem.* 41 (2005) 1–16.
- [3] Yongzhi Zhang, Rui Xiong, Hongwen He, Michael G. Pecht, Long short-term memory recurrent neural network for remaining useful life prediction of lithium-ion batteries, *IEEE Trans. Veh. Technol.* 67 (7) (2018) 5695–5705.
- [4] Xiaoyu Li, Lei Zhang, Zhenpo Wang, Peng Dong, Remaining useful life prediction for lithium-ion batteries based on a hybrid model combining the long short-term memory and Elman neural networks, *J. Energy Storage* 21 (2019) 510–518.
- [5] C. Su, H.J. Chen, A review on prognostics approaches for remaining useful life of lithium-ion battery, in: *IOP Conference Series: Earth and Environmental Science*, Vol. 93, IOP Publishing, 2017, 012040.
- [6] Alexander Bartlett, James Marcicki, Simona Onori, Giorgio Rizzoni, Xiao Guang Yang, Ted Miller, Electrochemical model-based state of charge and capacity estimation for a composite electrode lithium-ion battery, *IEEE Trans. Control Syst. Technol.* 24 (2) (2015) 384–399.
- [7] Anirudh Allam, Simona Onori, Online capacity estimation for lithium-ion battery cells via an electrochemical model-based adaptive interconnected observer, *IEEE Trans. Control Syst. Technol.* 29 (4) (2020) 1636–1651.
- [8] Michael Bosello, Carlo Falcomer, Claudio Rossi, Giovanni Pau, To charge or to sell? EV pack useful life estimation via LSTMs, CNNs, and autoencoders, *Energies* 16 (6) (2023) 2837.
- [9] Kailong Liu, Yunlong Shang, Quan Ouyang, Widanalege Dhammika Widanage, A data-driven approach with uncertainty quantification for predicting future capacities and remaining useful life of lithium-ion battery, *IEEE Trans. Ind. Electron.* 68 (4) (2020) 3170–3180.
- [10] Xin Lai, Wei Yi, Yifan Cui, Chao Qin, Xuebing Han, Tao Sun, Long Zhou, Yuejiu Zheng, Capacity estimation of lithium-ion cells by combining model-based and data-driven methods based on a sequential extended Kalman filter, *Energy* 216 (2021) 119233.
- [11] Gong Cheng, Xinzhi Wang, Yurong He, Remaining useful life and state of health prediction for lithium batteries based on empirical mode decomposition and a long and short memory neural network, *Energy* 232 (2021) 121022.
- [12] Maher Al-Greer, Imran Bashir, et al., Capacity estimation of lithium-ion batteries based on adaptive empirical wavelet transform and long short-term memory neural network, *J. Energy Storage* 70 (2023) 108046.
- [13] Fei Xia, Kangan Wang, Jiajun Chen, State of health and remaining useful life prediction of lithium-ion batteries based on a disturbance-free incremental capacity and differential voltage analysis method, *J. Energy Storage* 64 (2023) 107161.
- [14] Ashkan Lotfipoor, Sandhya Patidar, David P. Jenkins, Deep neural network with empirical mode decomposition and Bayesian optimisation for residential load forecasting, *Expert Syst. Appl.* 237 (2024) 121355.
- [15] Sinvaldo Rodrigues Moreno, Laio Oriel Seman, Stefano Frizzo Stefenon, Leandro dos Santos Coelho, Viviana Cocco Mariani, Enhancing wind speed forecasting through synergy of machine learning, singular spectral analysis, and variational mode decomposition, *Energy* 292 (2024) 130493.
- [16] Juncheng Bai, Jianfeng Guo, Bingzhen Sun, Yuqi Guo, Qiang Bao, Xia Xiao, Intelligent forecasting model of stock price using neighborhood rough set and multivariate empirical mode decomposition, *Eng. Appl. Artif. Intell.* 122 (2023) 106106.
- [17] Hang Zhao, Yujing Wang, Juanyong Duan, Congrui Huang, Defu Cao, Yunhai Tong, Bixiong Xu, Jing Bai, Jie Tong, Qi Zhang, Multivariate time-series anomaly detection via graph attention network, in: *2020 IEEE International Conference on Data Mining, ICDM, IEEE, 2020*, pp. 841–850.

- [18] Abhimanyu Das, Weihao Kong, Andrew Leach, Rajat Sen, Rose Yu, Long-term forecasting with TIDE: Time-series dense encoder, 2023, arXiv preprint arXiv:2304.08424.
- [19] Liaogehao Chen, Yong Zhang, Ying Zheng, Xiangshun Li, Xiujuan Zheng, Remaining useful life prediction of lithium-ion battery with optimal input sequence selection and error compensation, *Neurocomputing* 414 (2020) 245–254.
- [20] Hancheng Dong, Xiaoning Jin, Yangbing Lou, Changhong Wang, Lithium-ion battery state of health monitoring and remaining useful life prediction based on support vector regression-particle filter, *J. Power Sour.* 271 (2014) 114–123.
- [21] Zhiwei Xue, Yong Zhang, Cheng Cheng, Guijun Ma, Remaining useful life prediction of lithium-ion batteries with adaptive unscented kalman filter and optimized support vector regression, *Neurocomputing* 376 (2020) 95–102.
- [22] Jianfang Jia, Jianyu Liang, Yuanhao Shi, Jie Wen, Xiaoqiong Pang, Jianchao Zeng, SOH and RUL prediction of lithium-ion batteries based on Gaussian process regression with indirect health indicators, *Energies* 13 (2) (2020) 375.
- [23] Jian Liu, Ziqiang Chen, Remaining useful life prediction of lithium-ion batteries based on health indicator and Gaussian process regression model, *IEEE Access* 7 (2019) 39474–39484.
- [24] Chaolong Zhang, Shaishai Zhao, Yigang He, An integrated method of the future capacity and RUL prediction for lithium-ion battery pack, *IEEE Trans. Veh. Technol.* 71 (3) (2021) 2601–2613.
- [25] Kodjo SR Mawonou, Akram Eddahech, Didier Dumur, Dominique Beauvois, Emmanuel Godoy, State-of-health estimators coupled to a random forest approach for lithium-ion battery aging factor ranking, *J. Power Sources* 484 (2021) 229154.
- [26] Tadele Mamo, Fu-Kwun Wang, Attention-based long short-term memory recurrent neural network for capacity degradation of lithium-ion batteries, *Batteries* 7 (4) (2021) 66.
- [27] Fu-Kwun Wang, Zemen Endalamaw Amogne, Jia-Hong Chou, Cheng Tseng, Online remaining useful life prediction of lithium-ion batteries using bidirectional long short-term memory with attention mechanism, *Energy* 254 (2022) 124344.
- [28] Fei Xia, Kangan Wang, Jiajun Chen, State-of-health prediction for lithium-ion batteries based on complete ensemble empirical mode decomposition with adaptive noise-gate recurrent unit fusion model, *Energy Technol.* 10 (4) (2022) 2100767.
- [29] Ramon Gomes da Silva, Matheus Henrique Dal Molin Ribeiro, Sinvaldo Rodrigues Moreno, Viviana Cocco Mariani, Leandro dos Santos Coelho, A novel decomposition-ensemble learning framework for multi-step ahead wind energy forecasting, *Energy* 216 (2021) 119174.
- [30] Jiantao Qu, Feng Liu, Yuxiang Ma, Jiaming Fan, A neural-network-based method for RUL prediction and SOH monitoring of lithium-ion battery, *IEEE Access* 7 (2019) 87178–87191.
- [31] Xiaoqiong Pang, Rui Huang, Jie Wen, Yuanhao Shi, Jianfang Jia, Jianchao Zeng, A lithium-ion battery RUL prediction method considering the capacity regeneration phenomenon, *Energies* 12 (12) (2019) 2247.
- [32] Albert Gu, Karan Goel, Ankit Gupta, Christopher Ré, On the parameterization and initialization of diagonal state space models, *Adv. Neural Inf. Process. Syst.* 35 (2022) 35971–35983.
- [33] Albert Gu, Tri Dao, Mamba: Linear-time sequence modeling with selective state spaces, 2023, arXiv preprint arXiv:2312.00752.
- [34] Lianghui Zhu, Bencheng Liao, Qian Zhang, Xinlong Wang, Wenyu Liu, Xinggong Wang, Vision mamba: Efficient visual representation learning with bidirectional state space model, 2024, arXiv preprint arXiv:2401.09417.
- [35] Norden E Huang, Zheng Shen, Steven R Long, Manli C Wu, Hsing H Shih, Quanan Zheng, Nai-Chyuan Yen, Chi Chao Tung, Henry H Liu, The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, *Proc. R. Soc. Lond. Ser. A: Math. Phys. Eng. Sci.* 454 (1971) (1998) 903–995.
- [36] Kaiming He, Xiangyu Zhang, Shaoqing Ren, Jian Sun, Deep residual learning for image recognition, in: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2016, pp. 770–778.
- [37] Yu Zhang, Qiang Yang, An overview of multi-task learning, *Natl. Sci. Rev.* 5 (1) (2018) 30–43.
- [38] B. Sahaand, K. Goebel, Battery Data Set, NASA Ames Prognostics Data Repository, NASA Ames Research Center, 2007.
- [39] Kristen A Severson, Peter M Attia, Norman Jin, Nicholas Perkins, Benben Jiang, Zi Yang, Michael H Chen, Muratahan Aykol, Patrick K Herring, Dimitrios Fraggedakis, et al., Data-driven prediction of battery cycle life before capacity degradation, *Nat. Energy* 4 (5) (2019) 383–391.
- [40] Ailing Zeng, Muxi Chen, Lei Zhang, Qiang Xu, Are transformers effective for time series forecasting? in: *Proceedings of the AAAI Conference on Artificial Intelligence*, Vol. 37, 2023, pp. 11121–11128.